On the Complexity of Fuzzy Boolean Constraint Satisfaction Problems With Applications to Intelligent Digital Photography

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***Abstract*—We consider fuzzy Boolean constraint satisfaction problems, determine their complexity, isolate their islands of tractability, and show how they can be applied in digital photography.**

***Keywords*-Fuzzy Theory and Models, Fuzzy Mathematics and Applications**

I. Introduction

Started with Zadeh's seminal paper [14], *fuzzy sets* and *fuzzy logic* rapidly became an important part of logic, set theory and theoretical computer science with interesting industrial applications. Within these applications, the most important include *digital control*, especially in automotive and vehicle subsystems, in embarked systems, and in home appliances; *decision making*; *digital image processing* especially in digital cameras where it is applied both before (focusing) as well as after the actual photographic action; *video games*; *pattern recognition* and *machine vision* with the special emphasis on OCR; *telecommunications* with voice recognition systems; *medicine*; and last but not least *agriculture*. This list is far from from being complete and virtually all branches of applied artificial intelligence have a certain relationship to fuzzy logic [8]. Fuzzy logic, contrary to usual crisp logic, does not limit itself to two truth constants, usually denoted by 0 (or *false*) and 1 (or *true*), but it allows to use all logic values in the interval $[0, 1]$. In the same spirit, fuzzy sets contain elements with a certain degree of membership, expressing this way the affinity of each element with the property of the particular fuzzy set.

Zadeh's first approach to fuzzy logic uses a straightforward generalization of the usual logical connectives like conjunction, disjunction, and negation, as well as their set-theoretical counterparts of intersection, union, and complement. Since then many researchers generalized and enlarged these concepts to formal fuzzy logic, especially to the propositional fuzzy logic, as well as to formal fuzzy set theory [8], [9].

The study of constraint satisfaction problems (CSP) started in the 1970's and developed meanwhile to a full-fledged and important part of artificial intelligence and computer science [6]. It has been often mentioned that despite much research which has been done on constraint satisfaction problems, the results are not very satisfactory when applied to real-life problems. With the incorporation of the concept of fuzziness, fuzzy constraint satisfaction problems (FCSP) model real-life problems better by allowing individual constraints to be either fully or partially satisfied. This approach also allows to handle constraint satisfaction problems that appear to be inconsistent or overconstrained in the crisp setting. Probably the first attempt to formalize fuzzy constraint satisfaction problems has been done by Dubois et al. in [7], followed by Ruttkay in [12]. Fuzzy constraints and their satisfaction problems appear thereafter in plenty of papers, where the fuzzy constraint part is defined as an extension of usual relation-based constraints to fuzzy relations without any further theoretical consideration. No real complexity analysis of constraint satisfaction problems has been done, except to notice that the complexity of FCSP is not higher than the usual (crisp) CSP and that it is NP-complete since a crisp CSP is a special case of FCSP. Fuzzy constraint satisfaction problems can be also seen as a particular type of soft constraints as they were considered by Bistarelli et al [1], [2]. Other types of CSP related to fuzzy constraints are *weighted* CSP and *probabilistic* CSP. While fuzzy CSP associate a level of preference with each tuple in each constraint, in weighted CSP tuples come with an associated cost, allowing us to model optimization problems. Probabilistic CSP have been introduced to model situation where each constraint has a certain probability, allowing us to reason also about problems which are only partially known, contrary to fuzzy CSP where the problems are totally known but each model satisfies a constraint with a certain satisfaction level from the interval $[0, 1]$.

II. Preliminaries

We assume that the reader is familiar with the basics of fuzzy logic and fuzzy set theory, as well as with constraint satisfaction problems. The interested reader can find more information in the monographs [8]–[10] for fuzzy logic and in [3]–[6] for constraint satisfaction problems and their complexity. For complexity theory the interested reader can find more information in the monograph [11].

Given a universe U, which can be a finite or infinite set, a *fuzzy set* A over U is determined by its *membership function* $A\colon U \to [0, 1]$. The value $A(u)$ is called the *membership degree*. A fuzzy set is called *crisp* if $A(u) \in \{0, 1\}$ holds for each $u \in U$, i.e. if each element of a domain has a membership degree assigned to only one of the values 0 or 1. For convenience, we denote each element of a fuzzy set A by a tuple (a, α), where $a \in D$ and $A(a) = \alpha$. The implicit understanding of $(a, \alpha) \in A$ is that the element a belongs to the set A with the degree α. For each $\alpha \in [0, 1]$, an *α-cut* or an *α-level* of a fuzzy set A is the set $A|_\alpha = \{u \in U \mid A(u) \geq \alpha\}$.

I our setting, we will consider every construction over the Boolean domain $\{0, 1\}$. The universe U will be the Cartesian product $\{0, 1\}^k$ for some arity k and each element of a Boolean fuzzy set A will be a tuple (m, α), where $m \in \{0, 1\}^k$ is a Boolean vector and α is a real number from the interval $[0, 1]$. An *atomic Boolean fuzzy constraint* $A(x_1, \ldots, x_k)$ is an application of a Boolean fuzzy set A over the universe $\{0, 1\}^k$ with arity k to a variable vector $(x_1, \ldots, x_k)$ of the same arity, where each x_i belongs to a countably infinite set of variables X. A *Boolean fuzzy constraint* $\varphi(\vec{x})$ is (1) an atomic Boolean fuzzy constraint $A(\vec{x})$, or (2) a conjunction of two Boolean fuzzy constraints $\varphi_1(\vec{x}) \wedge \varphi_2(\vec{x})$, or (3) an existentially quantified fuzzy Boolean constraint $\exists y\ \varphi'(\vec{x}, y)$. We also say that Boolean fuzzy constraints are *primitive positive formulas* over atomic fuzzy Boolean constraints, according to the formulas built by conjunction $\wedge$ and existential quantification $\exists$.

The semantics of fuzzy Boolean constraints is defined by means of interpretations and level satisfaction. An *interpretation* is a mapping $m\colon X \to \{0,1\}$ of variables to the Boolean domain $\{0,1\}$. An interpretation m applied to a variable vector $(x_1,\ldots,x_k)$, denoted $m(x_1,\ldots,x_k)$, is the Boolean vector $(m(x_1),\ldots,m(x_k))$. If the variable vector $\vec{x}$ is known and the sequence of variables in it is fixed, we denote $m(\vec{x})$ by $(m_1,\ldots,m_k)$, where $m_i = m(x_i)$ for each i. We also overload the notation m, denoting both the interpretation and the Boolean vector $(m_1,\ldots,m_k)$. Instead of the vector $(m_1,\ldots,m_k)$ we often write only the string $m_1 \cdots m_k$ without loss of generality.

An interpretation m *α-satisfies* an atomic fuzzy Boolean constraint $A(x_1,\ldots,x_k)$ for an $\alpha \in [0,1]$, written $m \models_\alpha A(x_1,\ldots,x_k)$, if there exists an element $(m,\alpha') \in A$, such that $\alpha' \geq \alpha$. An interpretation m α-satisfies a fuzzy Boolean constraint $\varphi_1 \wedge \varphi_2$ if $m \models_\alpha \varphi_1$ and $m \models_\alpha \varphi_2$. An interpretation m α-satisfies a fuzzy Boolean constraint $\exists y\ \varphi(\vec{x}, y)$ if either $m0 \models_\alpha \varphi(\vec{x}, y)$ or $m1 \models_\alpha \varphi(\vec{x}, y)$, where $m0$ and $m1$ denote the extensions of m with 0 or 1, respectively. Of course, 0-satisfaction does not make sense, since then everything satisfies the constraint. For 1-satisfaction we also use the term *full satisfaction*, whereas for 0.5-satisfaction we coin the term *majority satisfaction*.

III. Boolean Fuzzy Constraint Satisfaction Problems

Let S be a set of Boolean fuzzy sets. The *Boolean fuzzy constraint satisfaction problem* FCSP is defined as follows.

Problem: BOOLEAN FCSP (S)
Input: A Boolean fuzzy constraint $\varphi(\vec{x})$ constructed over fuzzy sets from S and a value $\alpha \in [0,1]$.
Question: Is $\varphi(\vec{x})$ α-satisfiable?

Satisfaction of Boolean fuzzy constraint satisfaction problems is a more involved issue than in the case of crisp CSP, defined as follows.

Problem: BOOLEAN CSP (S)
Input: A Boolean constraint $\varphi(\vec{x})$ constructed over Boolean relations from S
Question: Is $\varphi(\vec{x})$ satisfiable?

The complexity changes arising with FCSP according to the choice of the threshold level α, compared with CSP where such changes are impossible, are well presented by the following example.

Example 1 Consider the FCSP constructed upon the fuzzy Boolean relation $R = \{(000,0),\ (001,1),\ (010,1),\ (011,0.25),\ (100,1),\ (101,0.25),\ (110,0.25),\ (111,0.5)\}$. If we ask if a Boolean fuzzy constraint φ, constructed upon the Boolean fuzzy relation R, is 0.75-satisfied then the problem becomes NP-complete, since the only Boolean vectors belonging to R with a degree $\alpha \geq 0.75$ are 001, 010, and 100. They constitute the relation called 1-IN-3. According to Schaefer [13], a CSP constructed upon the Boolean relation 1-IN-3 is NP-complete. However, if ask if a Boolean fuzzy constraint φ', constructed upon the Boolean fuzzy relation R, is 0.5-satisfied then the problem remains decidable in polynomial time. Indeed, the Boolean vectors belonging to R with a degree $\alpha \geq 0.5$ are 001, 010, 100, and 111. They constitute the relation represented by the solution set of the equation $x + y + z = 1 \pmod 2$, which is a linear system over the Boolean field $\mathbb{Z}_2$. Since linear systems over a field can be solved in polynomial time by Gaussian elimination, the 0.5-satisfiability of the FCSP over the relation R can be decided in polynomial time. Hence, depending on the threshold level α, a FCSP can be either NP-complete or polynomial-time decidable.

As in Example 1, we can transform each Boolean FCSP(S) with a threshold level α to a standard Boolean CSP$(S|_\alpha)$ where $S|_\alpha = \{R|_\alpha \mid R \in S\}$ is the set of α-cuts of the relations from S, as it has been proposed e.g. in [1]. The important issue here is that the complexity of the fuzzy Boolean constraint satisfaction problem can change not only according to the choice of the set of fuzzy relations S, but also according to the choice of the threshold level α.

Our goal will be to isolate island of tractability for fuzzy Boolean constraint satisfaction problems for all possible thresholds α. To do this, we must determine the types of FCSP which will remain decidable in polynomial time for every degree of satisfaction α. Schaefer identified in [13] six classes of Boolean constraint satisfaction problems decidable in polynomial time.

The first class of constraints decidable in polynomial time is built upon *Horn* relations [4]. We extend this notion to fuzzy Boolean relations. Horn relations are the solution sets of Horn formulas (conjunctions of Horn clauses — clauses with at most one positive literal) and are closed under conjunction, i.e. for a (crisp) Boolean relation B, for all Boolean vectors m and m', if $m \in B$ and $m' \in B$ then also $m \wedge m' \in B$. The conjunction $m \wedge m'$ is computed componentenwise, i.e. if $m = (m_1,\ldots,m_k)$ and $m' = (m'_1,\ldots,m'_k)$ then $m \wedge m' = (m_1 \wedge m'_1,\ldots,m_k \wedge m'_k)$. To maintain this closure property, we need to ensure for a fuzzy Horn relation R for all degrees γ, when $(m,\alpha) \in R$ and $(m',\alpha') \in R$ implies $(m \wedge m', \alpha'') \in R$, that $\alpha \geq \gamma$ and $\alpha' \geq \gamma$ implies $\alpha'' \geq \gamma$. The largest γ satisfying this property is $\gamma = \min\{\alpha, \alpha'\}$.

The second class of constraints decidable in polynomial time is built upon *dual Horn* relations [4]. Dual Horn relations are the solution sets of dual Horn formulas (conjunctions of dual Horn clauses — clauses with at most one negative literal) and are closed under disjunction, i.e. for a Boolean relation B if $m \in B$ and $m' \in B$ then also $m \vee m' \in B$, where the disjunction is computed componentenwise. To maintain this closure property, we need to ensure for a fuzzy Horn relation R for all degrees γ, when $(m,\alpha) \in R$ and $(m',\alpha') \in R$ implies $(m \vee m', \alpha'') \in R$, that $\alpha \geq \gamma$ or $\alpha' \geq \gamma$ implies $\alpha'' \geq \gamma$. The largest γ satisfying this property is $\gamma = \max\{\alpha, \alpha'\}$.

The third class of constraints decidable in polynomial time is built upon *bijunctive* relations [4]. Bijunctive relations are the solution sets of 2SAT formulas (conjunctions of clauses with at most two literals) and are closed under majority, a ternary associative-commutative Boolean operation defined by $\mathrm{maj}(0,0,1) = 0$, $\mathrm{maj}(0,1,1) = 1$, and $\mathrm{maj}(b,b,b) = b$ for each $b \in \{0,1\}$. Majority is a median operation a Boolean lattice, satisfying the identity $\mathrm{maj}(x,y,z) = (x \vee y) \wedge (y \vee z) \wedge (z \vee x)$. To maintain this closure property, we need to ensure for a fuzzy bijunctive relation R for all degrees γ, when $(m,\alpha), (m',\alpha'), (m'',\alpha'') \in R$ implies $(\mathrm{maj}(m,m',m''), \beta) \in R$, the following property: if at least two of the degrees α, α', and α'' are greater or equal to γ then this implies $\beta \geq \gamma$. The largest γ satisfying this property is $\gamma = \mathrm{med}(\alpha, \alpha', \alpha'')$, where med is the associative-commutative median operator defined on a totally ordered set by $\mathrm{med}(x,y,z) = y$ if $x \leq y \leq z$.

The forth class of constraints decidable in polynomial time is built upon *affine* relations [4]. Affine relations are the solution sets of linear equational systems over the Boolean field $\mathbb{Z}_2$, hence each affine relation is an affine space, and therefore they are closed under the affinity operation $\mathrm{aff}(x,y,z) = x + y + z \pmod 2$. The Boolean relation B, considered as an affine space, satisfies the property $m - m' + m'' \in B$ for each $m, m', m'' \in B$. Let R be a Boolean fuzzy relation, $R|_\alpha = \{m \mid R(m) \geq \alpha\}$ its α-cut, and $\hat{R}_\alpha = \{\beta \mid \exists m \in R|_\alpha, R(m) = \beta \geq \alpha\}$ its *fuzzy α-quotient*. If $R|_\alpha$ is an affine space, we need $\hat{R}_\alpha$ to be also an affine space isomorphic to $R|_\alpha$, where the isomorphism is expressed by the bijective function $f_\alpha \colon R|_\alpha \to \hat{R}_\alpha$ satisfying the identity $f_\alpha(m) = \beta$ for each m with $R(m) = \beta \geq \alpha$. This must hold for each $\alpha \in [0,1]$. Each α-cut $R|_\alpha$ is an affine subspace of the vector space $\{0,1\}^k$ where k is the arity of the Boolean vectors in R. Hence the fuzzy 0-quotient $\hat{R}_0 = \hat{R}$ must be isomorphic to the vector space $\{0,1\}^k$. This isomorphism is expressed by the function $f \colon \{0,1\}^k \to [0,1]$ satisfying the property $f(m) = R(m)$. Hence, a fuzzy relation is affine if its crisp restriction is affine and each α-cut R_α is an affine space. The last condition considerably limits the possibilities to construct an affine fuzzy constraint satisfaction problems.

The last two classes of constraints decidable in polynomial time are 0-valid and 1-valid [4]. A Boolean relation is 0-valid, respectively 1-valid if $0 \cdots 0 \in B$, respectively $1 \cdots 1 \in B$. To extend this to fuzzy Boolean constraint satisfaction problems, a fuzzy relation R is 0-valid, respectively 1-valid, if $R(0 \cdots 0) \geq \alpha$, respectively $R(1 \cdots 1) \geq \alpha$, holds for all α. This is satisfied only by $R(0 \cdots 0) = 1$, respectively $R(1 \cdots 1) = 1$.

All six conditions can be checked in polynomial time, hence it is decidable in polynomial time if a fuzzy Boolean relation belongs to one of the aforementioned classes. The test for fuzzy Horn and dual Horn relations is easily performed together with the test for closure under conjunction or disjunction, respectively. The test for fuzzy bijunctive relations can be also performed together with the test of closure under majority, since the median operation is compatible with it. The test for fuzzy 0-valid and 1-valid relations is a simple syntactical check. The most involved test is that for fuzzy affine relations. Following example shows how this test is performed.

Example 2 Let us modify the degrees of the fuzzy relation from Example 1, creating a new fuzzy relation $R' = \{(000, 0.25), (001, 0.5), (010, 0.5), (011, 0.25), (100, 0.75), (101, 0.25), (110, 0.25), (111, 0.8)\}$. Relation R' is not 1-valid, since $R'(111) = 0.8 \neq 1$. Let us analyze all possible α-cuts of R'. First we must divide the interval $[0,1]$ into disjoint chunks according to the degrees of membership in R'. For $\alpha \in [0, 0.25]$ we have $R'|_\alpha = \{0,1\}^3$, for $\alpha \in (0.25, 0.5]$ we have $R'|_\alpha = \{001, 010, 100, 111\}$, for $\alpha \in (0.5, 0.75]$ we have $R'|_\alpha = \{100, 111\}$, for $\alpha \in (0.75, 0.8]$ we have $R'|_\alpha = \{111\}$, finally for $\alpha \in (0.8, 1]$ we have $R'|_\alpha = \emptyset$. All these α-cuts are affine spaces, with $\emptyset$ being a trivial one, hence R' is an affine fuzzy relation.

A natural question arising now is to know whether we identified all possible polynomial-time decidable cases of FCSP. No other structural subcase based on a condition imposed on the part of Boolean relation is possible, since this would imply a new polynomial-time decidable case for crisp CSP. However, Schaefer proved in [13] that no other polynomial-time decidable cases exist. The only remaining possibility is therefore a case based on a different choice of the membership degrees. However, we systematically derived conditions on threshold degrees guaranteeing polynomial-time decidability, therefore counterexamples showing that no other possibilities exist are easily constructed. Example 1 is a counterexample for 0-valid, 1-valid, and affine fuzzy relations. The fuzzy relation $R_h = \{(000, 0.1), (001, 0.75), (010, 0.5), (100, 0.25)\}$ is a counterexample for the Horn case. The fuzzy relation $R_d = \{011, 0.75), (101, 0.5), (110, 0.25), (111, 0.1)\}$ is a counterexample for the dual Horn and bijunctive cases.

We can solve the FCSP(S) problem for each of the six polynomial-time decidable cases by first transforming S to $S|_\alpha$ and then solving the usual CSP$(S|_\alpha)$ problem by means of the corresponding method for deciding 0-valid, 1-valid, Horn, dual Horn, bijunctive, and affine Boolean constraints through satisfiability solving of the corresponding propositional formulas. Horn constraints are solved by unit resolution and propagation, i.e. a unit clause x or $\neg x$, corresponding to a unary Horn constraint $H(x)$, determines the logical value 0 or 1 of the variable. This value is propagated into the rest of the constraint, followed by a simplification according to the rules of propositional logic. If a contradiction is derived then the algorithm returns 0. When no more unary constraints exist and no contradiction was derived, the remaining variables are set to 0 and the algorithm returns the value 1. Dual Horn constraints are solved using the duality principle by transformation to Horn constraints. Indeed, if $\varphi(x_1, \ldots, x_n)$ is a dual Horn constraint then $\neg\varphi(\neg x_1, \ldots, \neg x_n)$ is a Horn constraint. Bijunctive constraints, corresponding to conjunctions of bijunctive clauses, are solved by saturation by means of binary resolution, factorization, and unit constraint propagation. Binary resolution does not explode the length of clauses. Indeed, a resolution step between $(l \vee x)$ and $(\neg x \vee l')$ produces the new clause $l \vee l'$. Hence, there is only a quadratic number of possible bijunctive clauses, corresponding to atomic bijunctive constraints, over a given set of variables. Affine constraints correspond to linear systems over the Boolean field $\mathbb{Z}_2$. They are solved by gaussian elimination. Finally, 0-valid and 1-valid constraints are solved by assigning the logical value 0, respectively 1, to all variables.

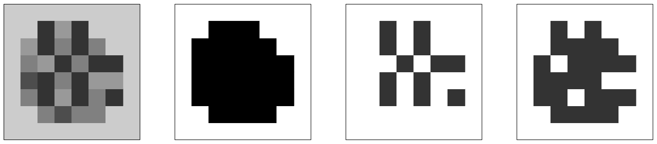
IV. Intelligent Digital Photography

Up to date intelligent photography uses fuzzy in *auto focus* (AF) and *auto exposure* (AE).

Exposure is determined by a combination of shutter speed, aperture (f), and CMOS[1] speed (ISO). For any given situation, there are many 'creatively correct' exposures, usually classified in the following kinds: (a) Story telling — small apertures (f/32, f/22, f/16); (b) Isolation — large apertures (f/2.8, f/4, f/5.6); (c) Who cares — in between apertures, depth of field is not a concern; (d) Macro — out of focus background (bokeh); (e) Freeze action — fast shutter speeds (1/1000, 1/500, 1/250); (f) Panning — medium shutter speeds (1/60, 1/30); (g) Imply motion — snow shutter speeds (1/4, 1/2, 1). Choosing the 'correct' exposure also depends on the metering technique (spot, center weighted, average, partial, evaluative, and scene modes). Auto exposure in a digital camera works as follows: (1) Exposure mode measures incoming light. (2) Calculates exposure so that scene is approximately 18% gray. (3) Calculates minimum shutter speed based on lens focal length. (4) Calculates required aperture for minimum shutter speed. (5) If not possible, add stops from ISO. (6) If necessary, shoot with aperture wide open.

Let us consider an 8×8 pixel square, which is a compressing unit of a JPEG image. This basic pixel square will be our relation of arity 64. The original picture is first passed through a filer with a threshold τ corresponding to the actual combination of shutter speed,

[1]Complementary Metal-Oxyde-Semiconductor digital recorder.

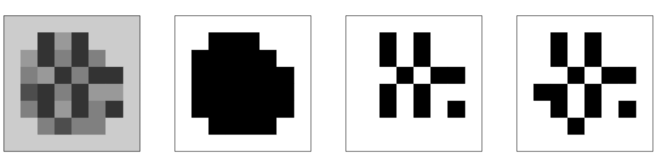


(a) Original (b) Too dark (c) Washed out (d) Excellent

Fig. 1: Exposure

aperture, and ISO speed. Every brightness below τ is interpreted as 0, whereas every brightness equal or above is interpreted as 1. This way the basic pixel square image becomes a Boolean vector of arity 64. The corresponding relation E (for exposure) cannot be known at the beginning, but is computed on the fly, according to the signals generated by the CMOS. The membership α degree of each element $(m, \alpha) \in E$ is determined by structural similarity with the original image: the more similar the vector m is with the original picture, the higher the membership degree α. The basic idea is to average the brightness, where the mean value is 18% gray. This averaging operation perfectly corresponds to the median operator, therefore the exposure relation has the best correspondence with the reality when it is bijunctive. Figure 1 visualizes well this situation. The original picture (1a) is first transformed to the Boolean vector (1b), which is too dark. Shutter speed and aperture is moved, so that the picture is transformed to the vector (1c), which is washed out. Finally, the right exposure is found with the vector (1d). The exposure of the whole picture is then determined as by a conjunction of the exposure constraints spanning the selected important parts of the picture, according to the selected metering technique. Hence, the exposure problem in digital photography can be formalized by means of fuzzy bijunctive constraints decidable in polynomial time, what makes that problem tractable.

Auto focus in digital photography is works as follows: (1) Looks at current image; (2) Move lens slightly, compare the two images (sharpness, contrast); (3) If better, keep moving. if worse, go other direction. Image is compared as lens focuses to choose the 'best' image using a sharpness measure over a portion of the image. Let us consider once more our 8×8 pixel square. The original picture is passed through a filter with a threshold τ corresponding to the actual contrast. If the contrast between two neighboring pixels is below τ, assign to both pixels the same Boolean value. If the contrast between them is equal or above τ, assign different Boolean values to the neighboring pixels. The corresponding relation F (for focus) is once more computed on the fly. The membership degree α of each element $(m, \alpha) \in F$ is determined by the number of polarity changes from 0 to 1 in the relation. Polarity changes are best expressed by the bijunctive affine relation generated as the solution set of the equation $x + y = 1 \pmod 2$. However, the number of such equations even for a 8×8 square would be too large. We can however express the whole basic pixel square as through a single equation $x_{11} + \cdots + x_{ij} + \cdots + x_{88} = 1$ for $i, j \in \{1, \ldots, 8\}$, where the elements (m, α) with a non-zero membership degree must be within the affine hull of the pixels with a predefined brightness. Figure 2 visualizes well this situation. For the original picture (2a) we compute first the affine hull (2b). We select the darkest points within this hull and compute the contrast, arriving at the shortest focus (2c). The camera now turns the lens, until it arrives at the picture with the largest contrast change (2d), corresponding to the vector with highest membership degree, which expresses the best focus.



(a) Original (b) Hull (c) Short (d) Right

Fig. 2: Focus

V. Concluding Remarks

We considered fuzzy Boolean constraint satisfaction problems, isolated the islands of tractability which correspond to polynomial-time decidable cases. Finally we showed how we can use these tractable cases of fuzzy Boolean constraint satisfaction problems for fast solving of the practical problems to compute auto exposure and auto focus in digital cameras.

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